# FUNCTIONS

Practice Questions (2015-2018)

0606 Revision Hub

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## 1. 0606/11 October/November 2018 Qn.11

**(a)** $f(x) = 3 - \cos 2x$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

**(i)** Write down the range of f. [2]

**(ii)** Find the exact value of $f^{-1}(2.5)$. [3]

**(b)** $g(x) = 3 - x^2$ for $x \in \mathbb{R}$.

Find the exact solutions of $g^2(x) = -6$. [4]

## 2. 0606/13 October/November 2018 Qn. 8

$$f(x) = 5 + \sin\frac{x}{4} \quad \text{for} \quad 0 \leqslant x \leqslant 2\pi \text{ radians}$$

$$g(x) = x - \frac{\pi}{3} \quad \text{for} \quad x \in \mathbb{R}$$

**(i)** Write down the range of $f(x)$. [2]

**(ii)** Find $f^{-1}(x)$ and write down its range. [3]

**(iii)** Solve $2fg(x) = 11$. [4]

## 3. 0606/22 October/November 2018 Qn. 11

The functions f and g are defined for real values of $x \geqslant 1$ by

$$f(x) = 4x - 3,$$

$$g(x) = \frac{2x+1}{3x-1}.$$

**(i)** Find $gf(x)$. [2]

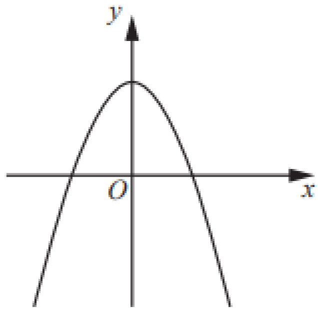
**(ii)** Find $g^{-1}(x)$. [3]

**(iii)** Solve $fg(x) = x - 1$. [4]

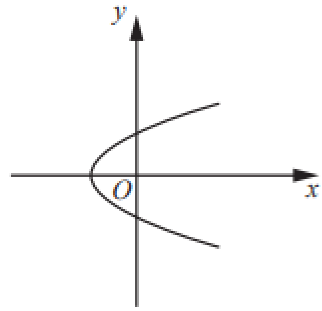
## 4. 0606/11 May/June 2018 Qn. 3

Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.

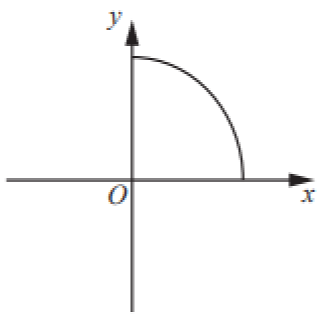
**A**



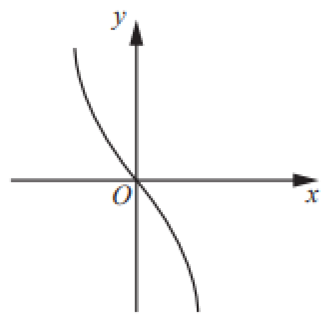

**B**




**C**




**D**




Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

| | **A** | **B** | **C** | **D** |
|---|---|---|---|---|
| Not a function | | | | |
| One-one function | | | | |
| A function that is its own inverse | | | | |
| A function with no inverse | | | | |

5. 0606/21 May/June 2018 Qn. 5

The function f is defined by $\mathrm{f}(x) = \dfrac{1}{2x-5}$ for $x > 2.5$.

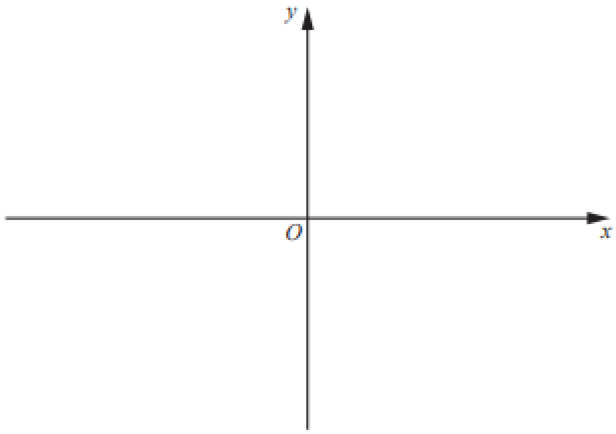
**(i)** Find an expression for $\mathrm{f}^{-1}(x)$. [2]

**(ii)** State the domain of $\mathrm{f}^{-1}(x)$. [1]

**(iii)** Find an expression for $\mathrm{f}^{2}(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where $a$, $b$, $c$ and $d$ are integers to be found. [3]

## 6. 0606/22 May/June 2018 Qn. 10

**(a)** **(i)** On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the $x$-axis. [2]




**(ii)** Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

**(b)** The functions g and h are defined by

$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

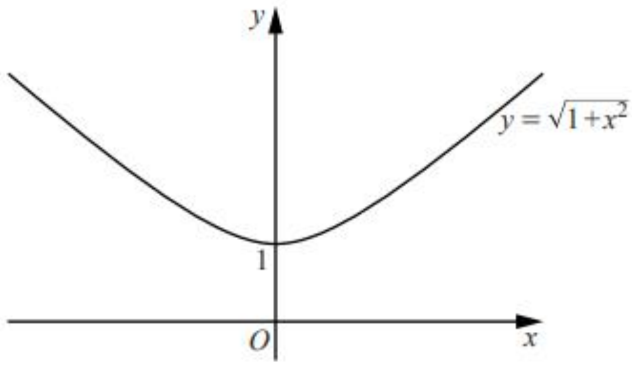
**(i)** Find $hg(x)$. [1]

**(ii)** Find $(hg)^{-1}(x)$. [2]

**(c)** Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

## 7. 0606/22 February/March 2018 Qn. 10

**(a)** The function f is defined by $f(x) = \sqrt{1 + x^2}$, for all real values of $x$. The graph of $y = f(x)$ is given below.




**(i)** Explain, with reference to the graph, why f does not have an inverse. [1]

**(ii)** Find $f^2(x)$. [2]

**(b)** The function g is defined, for $x > k$, by $g(x) = \sqrt{1 + x^2}$ and g has an inverse.

**(i)** Write down a possible value for $k$. [1]

**(ii)** Find $g^{-1}(x)$. [2]

## 8. 0606/11 October/November 2017 Qn. 6

**(a)** Functions f and g are such that, for $x \in \mathbb{R}$,

$$f(x) = x^2 + 3,$$

$$g(x) = 4x - 1.$$

**(i)** State the range of f. [1]

**(ii)** Solve $fg(x) = 4$. [3]

**(b)** A function h is such that $h(x) = \dfrac{2x+1}{x-4}$ for $x \in \mathbb{R}$, $x \neq 4$.

**(i)** Find $h^{-1}(x)$ and state its range. [4]

**(ii)** Find $h^2(x)$, giving your answer in its simplest form. [3]

## 9. 0606/12 October/November 2017 Qn. 6

Functions f and g are defined, for $x > 0$, by

$$f(x) = \ln x,$$
$$g(x) = 2x^2 + 3.$$

**(i)** Write down the range of f. [1]

**(ii)** Write down the range of g. [1]

**(iii)** Find the exact value of $f^{-1}g(4)$. [2]

**(iv)** Find $g^{-1}(x)$ and state its domain. [3]

10. 0606/23 October/November 2017 Qn. 6

The functions f and g are defined for real values of $x$ by

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1},\ x \neq \frac{1}{2}.$$

**(i)** Find $f^2(-3)$. [2]

**(ii)** Show that $g^{-1}(x) = g(x)$. [3]

**(iii)** Solve $gf(x) = \frac{8}{19}$. [4]

## 11. 0606/11 May/June 2017 Qn. 4

**(a)** It is given that $f(x) = 3e^{-4x} + 5$ for $x \in \mathbb{R}$.

**(i)** State the range of f. [1]

**(ii)** Find $f^{-1}$ and state its domain. [4]

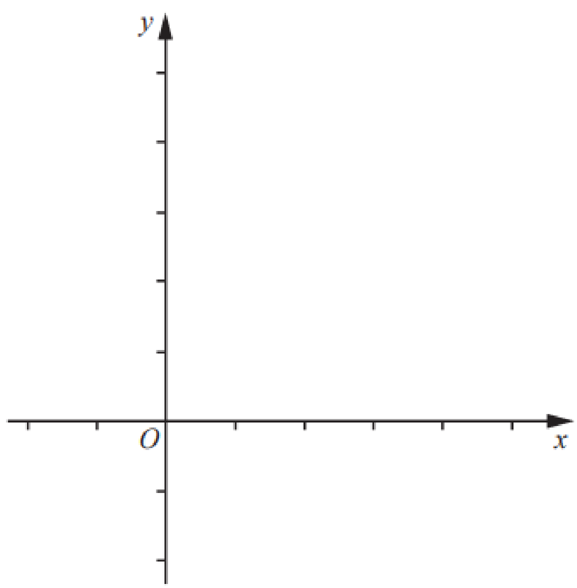
**(b)** It is given that $g(x) = x^2 + 5$ and $h(x) = \ln x$ for $x > 0$. Solve $hg(x) = 2$. [3]

## 12. 0606/22 May/June 2017 Qn. 9

A function f is defined, for $x \leqslant \frac{3}{2}$, by $\quad f(x) = 2x^2 - 6x + 5$.

**(i)** Express $f(x)$ in the form $a(x-b)^2 + c$, where $a$, $b$ and $c$ are constants. [3]

**(ii)** On the same axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the geometrical relationship between them. [3]




**(iii)** Using your answer from part **(i)**, find an expression for $f^{-1}(x)$, stating its domain. [3]

## 13. 0606/22 May/June 2017 Qn. 12

The function g is defined, for $x > -\frac{1}{2}$, by $g(x) = \frac{3}{2x+1}$.

**(i)** Show that $g'(x)$ is always negative. [2]

**(ii)** Write down the range of g. [1]

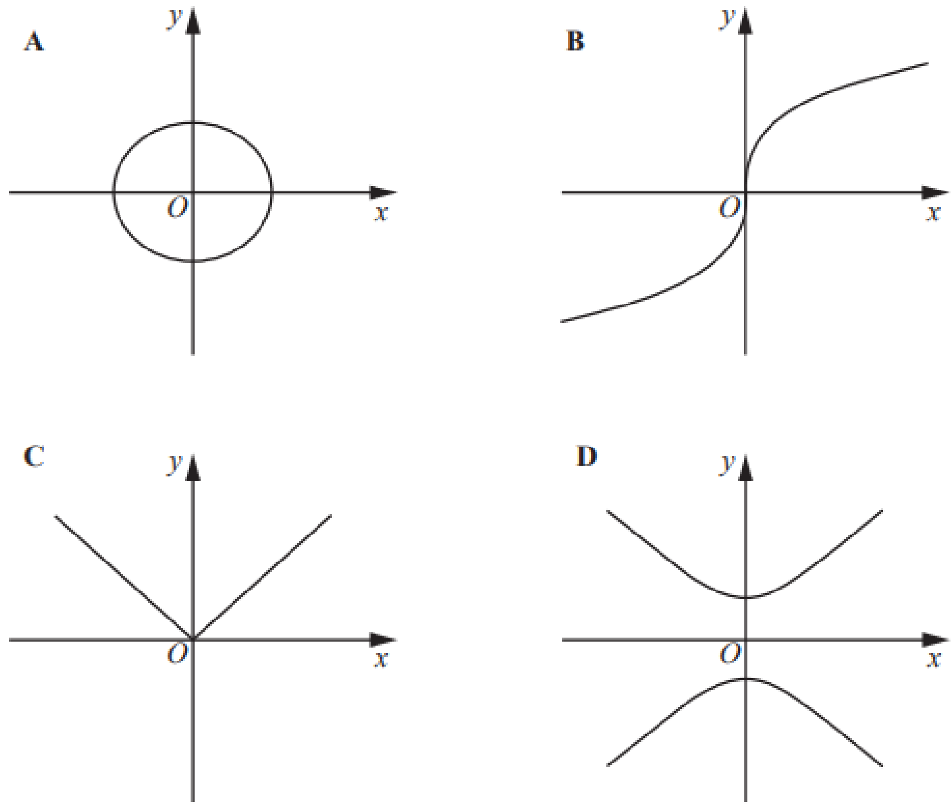
The function h is defined, for all real $x$, by $h(x) = kx + 3$, where $k$ is a constant.

**(iii)** Find an expression for $hg(x)$. [1]

**(iv)** Given that $hg(0) = 5$, find the value of $k$. [2]

**(v)** State the domain of hg. [1]

## 14. 0606/23 May/June 2017 Qn. 2




The four graphs above are labelled **A**, **B**, **C** and **D**.

**(i)** Write down the letter of each graph that represents a function, giving a reason for your choice. [2]

**(ii)** Write down the letter of each graph that represents a function which has an inverse, giving a reason for your choice. [2]

## 15. 0606/22 February/March 2017 Qn. 11

The functions f and g are defined by

$$f(x) = \frac{x^2 - 2}{x} \quad \text{for } x \geqslant 2,$$

$$g(x) = \frac{x^2 - 1}{2} \quad \text{for } x \geqslant 0.$$

**(i)** State the range of g. [1]

**(ii)** Explain why fg(1) does not exist. [2]

**(iii)** Show that $gf(x) = ax^2 + b + \frac{c}{x^2}$, where $a$, $b$ and $c$ are constants to be found. [3]

**(iv)** State the domain of gf. [1]

**(v)** Show that $f^{-1}(x) = \frac{x + \sqrt{x^2 + 8}}{2}$. [4]

## 16. 0606/23 October/November 2016 Qn. 10

The functions f and g are defined for $x > 1$ by

$$f(x) = 2 + \ln x,$$

$$g(x) = 2e^x + 3.$$

**(i)** Find $fg(x)$. [1]

**(ii)** Find $ff(x)$. [1]

**(iii)** Find $g^{-1}(x)$. [2]



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**(iv)** Solve the equation $f(x) = 4$. [1]

**(v)** Solve the equation $gf(x) = 20$. [4]



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17. 0606/11 May/June 2016 Qn. 6

The function f is defined by $f(x) = 2 - \sqrt{x+5}$ for $-5 \leqslant x < 0$.

**(i)** Write down the range of f. [2]

**(ii)** Find $f^{-1}(x)$ and state its domain and range. [4]

The function g is defined by $g(x) = \frac{4}{x}$ for $-5 \leqslant x < -1$.

**(iii)** Solve $fg(x) = 0$. [3]



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18. 0606/22 May/June 2016 Qn. 11

**(a)** A function f is defined, for all real $x$, by

$$f(x) = x - x^2.$$

Find the greatest value of $f(x)$ and the value of $x$ for which this occurs. [3]

**(b)** The domain of $g(x) = x - x^2$ is such that $g^{-1}(x)$ exists. Explain why $x \geqslant 1$ is a suitable domain for $g(x)$. [1]

**(c)** The functions h and k are defined by

$$h : x \mapsto \lg(x+2) \quad \text{for } x > -2,$$

$$k : x \mapsto 5 + \sqrt{x-1} \quad \text{for } 1 < x < 101.$$

**(i)** Find hk(10). [2]

**(ii)** Find $k^{-1}(x)$, stating its domain and range. [5]



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19. 0606/12 February/March 2016 Qn. 6

A function f is such that $f(x) = 6 + e^{4x}$ for $x \in \mathbb{R}$.

**(i)** Write down the range of f. [1]

**(ii)** Find $f^{-1}(x)$ and state its domain and range. [4]

**(iii)** Find $f'(x)$. [1]

**(iv)** Hence find the exact solution of $f(x) = f'(x)$. [2]



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## 20. 0606/11 October/November 2015 Qn. 11

**(a)** A function f is such that $f(x) = x^2 + 6x + 4$ for $x \geqslant 0$.

**(i)** Show that $x^2 + 6x + 4$ can be written in the form $(x + a)^2 + b$, where $a$ and $b$ are integers. [2]

**(ii)** Write down the range of f. [1]

**(iii)** Find $f^{-1}$ and state its domain. [3]

**(b)** Functions g and h are such that, for $x \in \mathbb{R}$,

$$g(x) = e^x \quad \text{and} \quad h(x) = 5x + 2.$$

Solve $h^2g(x) = 37$. [4]

## 21. 0606/23 October/November 2015 Qn. 9

Given that $f(x) = 3x^2 + 12x + 2$,

**(i)** find values of $a$, $b$ and $c$ such that $f(x) = a(x+b)^2 + c$, [3]

**(ii)** state the minimum value of $f(x)$ and the value of $x$ at which it occurs, [2]

**(iii)** solve $f\left(\frac{1}{y}\right) = 0$, giving each answer for $y$ correct to 2 decimal places. [3]

22. 0606/11 May/June 2015 Qn. 8

It is given that $f(x) = 3e^{2x}$ for $x \geqslant 0$,

$g(x) = (x+2)^2 + 5$ for $x \geqslant 0$.

**(i)** Write down the range of f and of g. [2]

**(ii)** Find $g^{-1}$, stating its domain. [3]

**(iii)** Find the exact solution of $gf(x) = 41$. [4]



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**(iv)** Evaluate $f'(\ln 4)$. [2]



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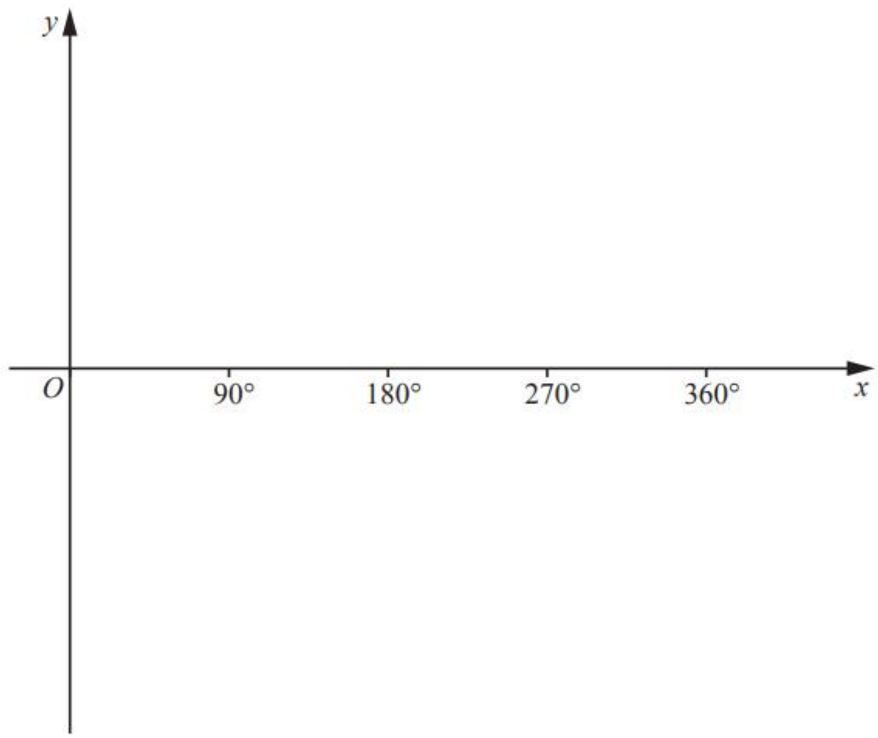
## 23. 0606/22 May/June 2015 Qn. 10

**(a)** The function f is defined by $f:x \mapsto |\sin x|$ for $0° \leqslant x \leqslant 360°$. On the axes below, sketch the graph of $y = f(x)$. [2]




**(b)** The functions g and hg are defined, for $x \geqslant 1$, by

$$g(x) = \ln(4x - 3),$$
$$hg(x) = x.$$

**(i)** Show that $h(x) = \dfrac{e^x + 3}{4}$. [2]



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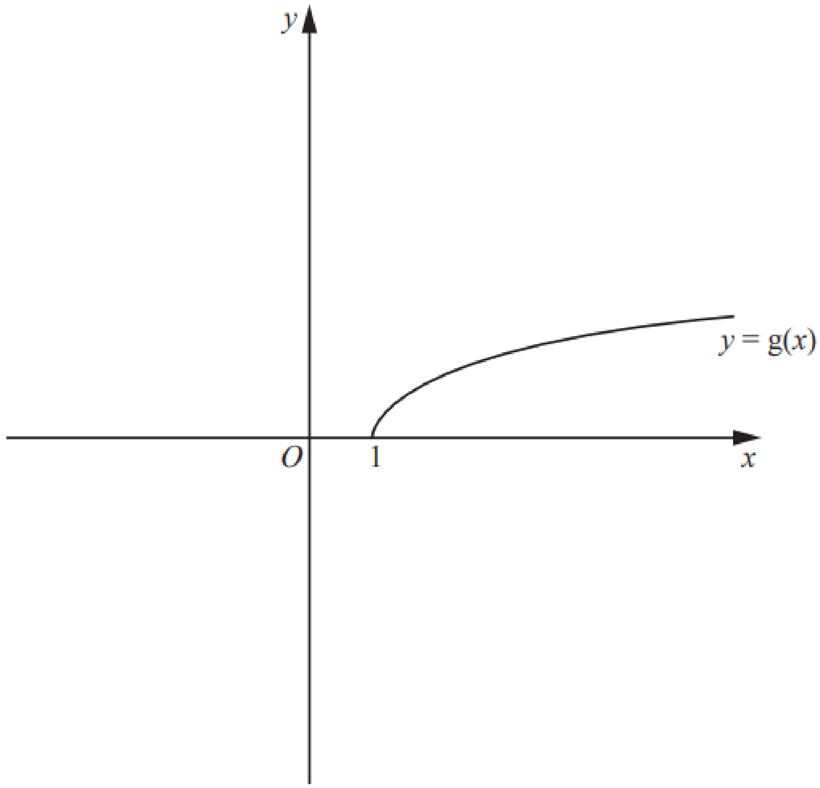
**(ii)**




The diagram shows the graph of $y = \mathrm{g}(x)$. Given that g and h are inverse functions, sketch, on the same diagram, the graph of $y = \mathrm{h}(x)$. Give the coordinates of any point where your graph meets the coordinate axes. [2]

**(iii)** State the domain of h. [1]

**(iv)** State the range of h. [1]

24. 0606/12 February/March 2015 Qn. 8

**(a)** A function f is such that $f(\theta) = \sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

**(i)** Write down the range of f. [1]

**(ii)** Write down a suitable restricted domain for f such that $f^{-1}$ exists. [1]

**(b)** Functions g and h are such that

$$g(x) = 2 + 4\ln x \text{ for } x > 0,$$
$$h(x) = x^2 + 4 \text{ for } x > 0.$$

**(i)** Find $g^{-1}$, stating its domain and its range. [4]

**(ii)** Solve $gh(x) = 10$. [3]



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**(iii)** Solve $g'(x) = h'(x)$. [3]



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